Find $\frac{d}{dx} x^3 \sinh^{-1}(x^2)$. Simplify your final answer. **SCORE: _____ / 3 PTS**

You may use the derivatives of any hyperbolic or inverse hyperbolic functions from your textbook without proving them.

$$3x^2 \sinh^{-1} x^2 + x^3 \cdot \frac{1}{\sqrt{1+(x^2)^2}} \, 2x$$

$$= \underbrace{3x^2 \sinh^{-1} x^2}_{(1)} + \underbrace{\frac{2x^4}{\sqrt{1+x^4}}}_{(1)\ (1)}$$

Find $\lim_{x\to-\infty} \coth x$. **Do NOT use a graph. Give *BRIEF* algebraic or numerical reasoning.** **SCORE: _____ / 3 PTS**

$$= \lim_{x\to-\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

(1) $= \lim_{x\to-\infty} \frac{e^{2x}+1}{e^{2x}-1}$

(1) $= \frac{0+1}{0-1}$

(1) $= -1$

OR

$= \lim_{x\to-\infty} \frac{1+e^{-2x}}{1-e^{-2x}}$ (1) $\frac{\infty}{-\infty}$

$= \lim_{x\to-\infty} \frac{-2e^{-2x}}{2e^{-2x}}$ (1)

$= -1$ (1)

☆ GRADE AGAINST ONE VERSION ONLY

If $\tanh x = -\frac{1}{3}$, find $\sinh x$.

SCORE: _____ / 4 PTS

You may use any hyperbolic identities from your textbook without proving them.

$1 - \tanh^2 x = \operatorname{sech}^2 x$

$1 - \frac{1}{9} = \operatorname{sech}^2 x$

(1) $\operatorname{sech}^2 x = \frac{8}{9}$

(1) $\operatorname{sech} x = \frac{2\sqrt{2}}{3}$ SINCE $\operatorname{sech} x > 0$ FOR ALL x

(1) $\cosh x = \frac{3}{2\sqrt{2}}$

$\tanh x = \frac{\sinh x}{\cosh x} \longrightarrow \sinh x = \tanh x \cosh x$

$= -\frac{1}{3} \cdot \frac{3}{2\sqrt{2}} =$ (1) $\frac{-1}{2\sqrt{2}}$ OR $-\frac{\sqrt{2}}{4}$

EITHER ANSWER OK

Prove the logarithmic formula for $\sinh^{-1} x$ given in your textbook. **SCORE: _____ / 4 PTS**

NOTE: This is NOT a question about derivatives.

$y = \sinh^{-1} x$

$\sinh y = x$

$\frac{e^y - e^{-y}}{2} = x$ (1/2)

LET $z = e^y$

$\frac{z - \frac{1}{z}}{2} = x$

$z - \frac{1}{z} = 2x$

(1) $z^2 - 1 = 2xz$

$z^2 - 2xz - 1 = 0$

$z = \frac{2x \pm \sqrt{4x^2 + 4}}{2}$ (1)

$z = \frac{2x \pm 2\sqrt{x^2+1}}{2}$

$e^y = x \pm \sqrt{x^2+1}$ (1/2) $x - \sqrt{x^2+1} < 0$

$e^y = x + \sqrt{x^2+1}$ (1/2)

$y = \ln(x + \sqrt{x^2+1}) = \sinh^{-1} x$ (1/2)

OK IF YOU USED e^y DIRECTLY INSTEAD OF z

For the following questions, you may use the formulae for $\frac{d}{dx}\sinh x$ and/or $\frac{d}{dx}\cosh x$ without proving them. **SCORE:** _____ / 6 PTS

If you need to use the formula for the derivative of any other hyperbolic function, you must prove it.

[a] **Without using the logarithmic formula for $\cosh^{-1} x$**, prove the formula for $\frac{d}{dx}\cosh^{-1} x$.

$y = \cosh^{-1} x$

$\cosh y = x$

(1) $(\sinh y)\frac{dy}{dx} = 1$

(1) $\frac{dy}{dx} = \frac{1}{\sinh y}$

$= \frac{1}{\sqrt{x^2 - 1}}$ (1/2)

$\cosh^2 y - \sinh^2 y = 1$

(1/2) $x^2 - \sinh^2 y = 1$

$\sinh^2 y = x^2 - 1$

BONUS (1) $\sinh y = \pm\sqrt{x^2 - 1}$

GRAPH OF $\cosh x$ IS 1-1 IF DOMAIN RESTRICTED TO $x \geq 0$

SO RANGE OF $\cosh^{-1} x$ IS $y \geq 0$

SO $\sinh y \geq 0$

[b] Prove the formula for $\frac{d}{dx}\coth x$.

$\frac{d}{dx}\frac{\cosh x}{\sinh x}$

$= \frac{(\sinh x)(\sinh x) - (\cosh x)(\cosh x)}{\sinh^2 x}$ (1)

$= \frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x}$ (1/2) $= \frac{-1}{\sinh^2 x}$ (1) $= -\operatorname{csch}^2 x$ (1/2)